SIMULATION TOOLS FOR STATISTICAL MODEL COMPARISON: AN APPLICATION TO UNOBSERVED COMPONENT MODELS VERSUS DYNAMIC REGRESSION MODELS

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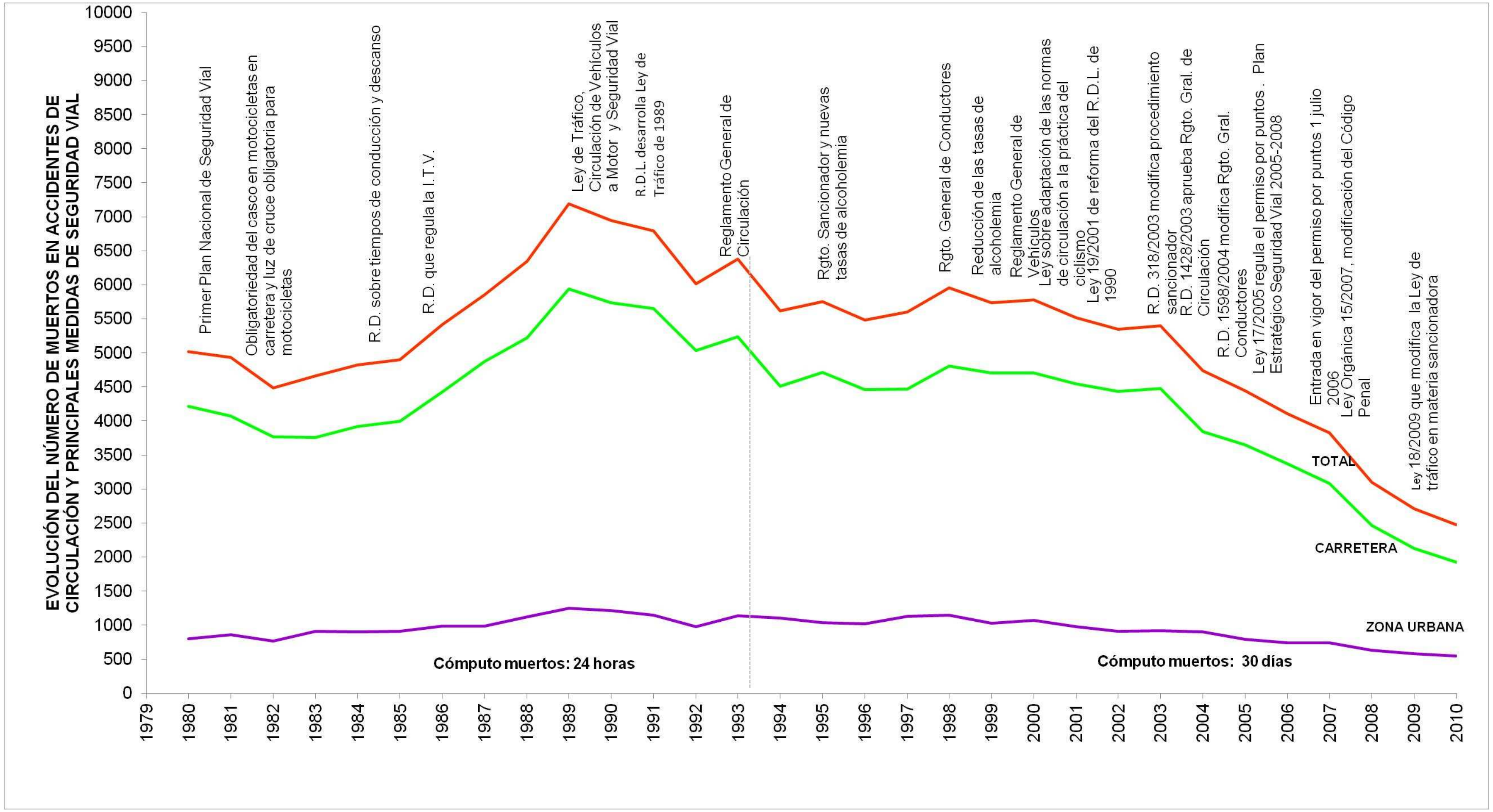
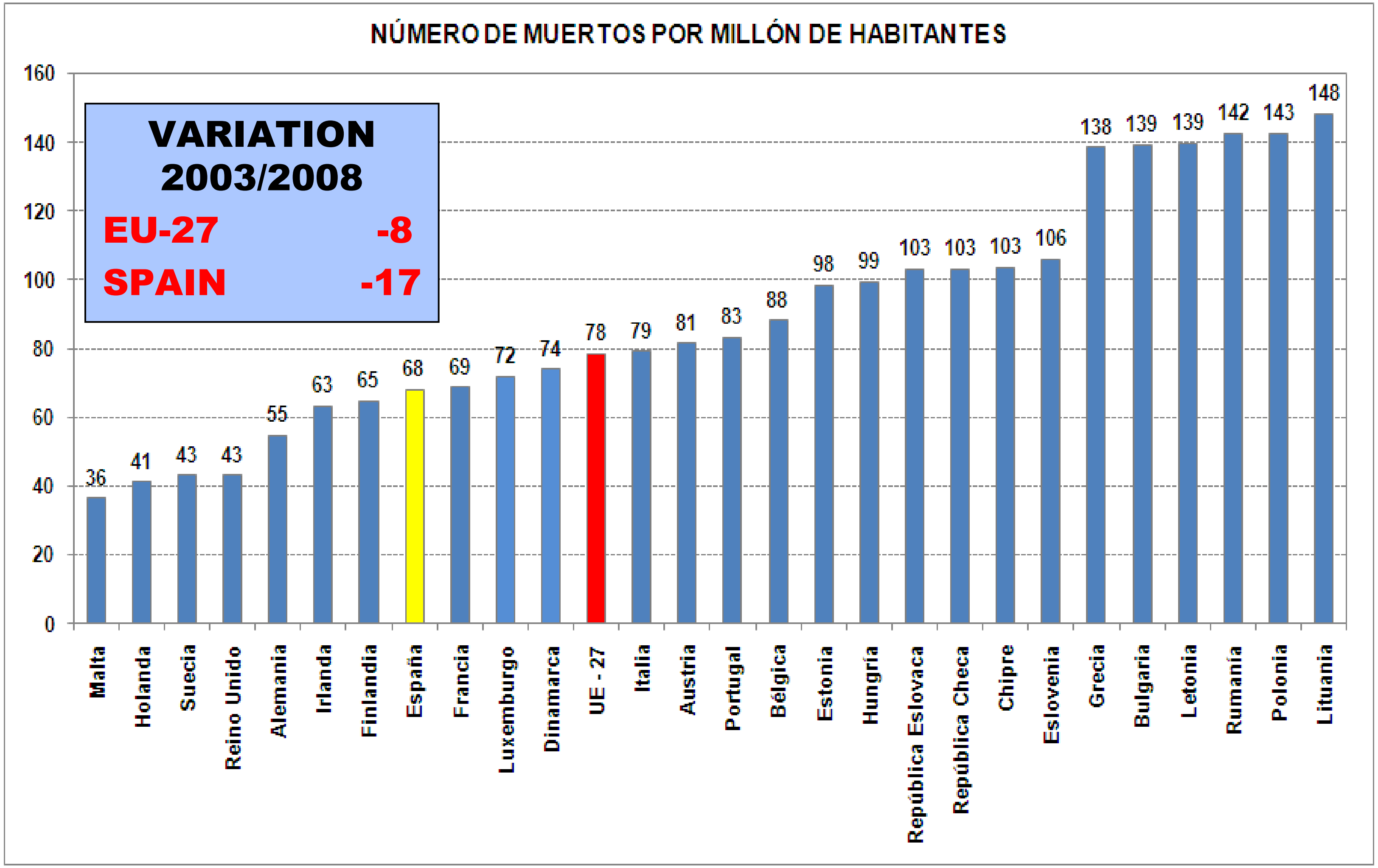
ROAD ACCIDENTS IN SPAIN

FATAL ITY RATE IN SPANISH ROADS-LEGISLATIVE CHANGES



ROAD ACCIDENTS IN SPAIN



DYNAMIC MACRO MODELS FOR ROAD ACCIDENT ANALYSIS

- Tool to quantify the effects mentioned above mainly two time series model are used:
  - *Demand for road use, accidents and their gravity* (DRAG), developed by Gaudry (1984) and Gaudry and Lassarre (2000) ;
  - *Unobserved Components Models* (UCM) with intervention proposed by Harvey and Durbin (1986).

- *Main differences*
- UCM includes unobserved specific terms for *trend* and *seasonality*
- State equations for both terms
- UCM more complex and general
- DRAG has simpler interpretation

Question:

Relationship between two models, which DRAG terms capture UC trend and seasonality?

PURPOSE

- Better understanding of relationship between the two “competing” models
- We suppose UC is the true model and:

  - ✓ See how the DRAG parameter estimates capture the UC terms.
  - ✓ Relationship between parameters of both models.
  - ✓ Eventually effect on prediction as well.

STAGES OF COMPUTATIONAL EXPERIMENT

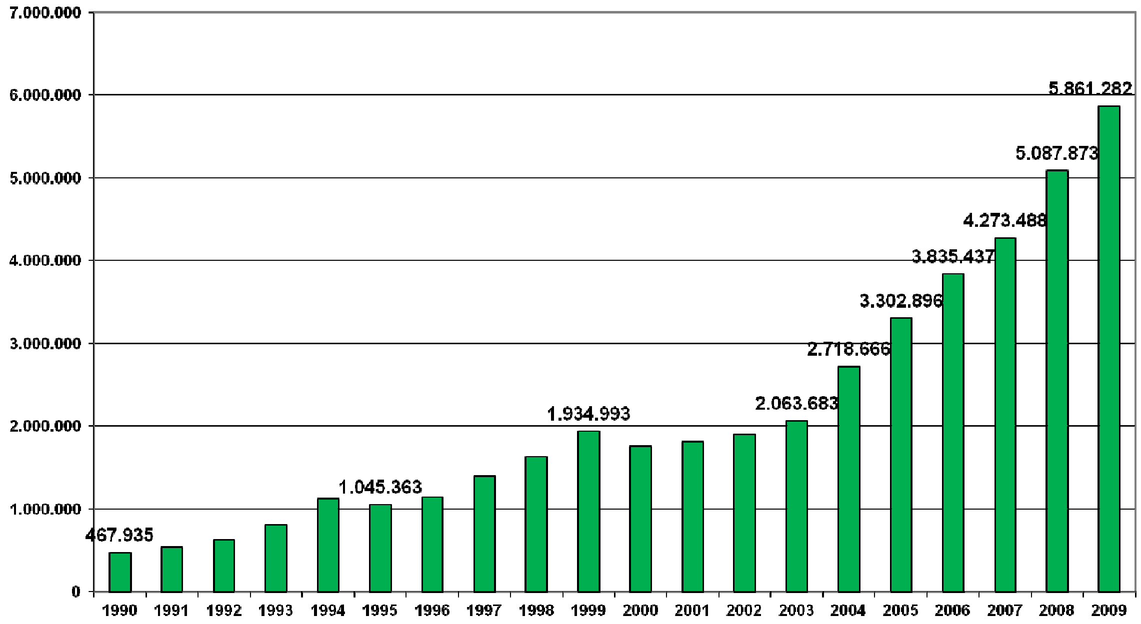
- The experiment is a **simulation** study with the following steps:
  - ✓ *Designing the simulations of the UC model using the results of the empirical study*
  - ✓ *Generation of UC samples (time series)*
  - ✓ *DRAG estimation:*
  - ✓ *ANOVA-type analysis of results*

INPUT VARIABLES

Table 1. Input variables

VARIABLES		
Exposure	Total fuel consumption	***COMTOT***
Economic factors	Unemployment rate in service sector	***PARSER***
Driver behavior	Number of alcohol control	***CONALC***
	Driver licensce suspencion	***SUSP***
Labor conditions	Number of labor days	***DLAB***

Number of alcohol controls (***CONALC***), **1990-2009**



STAGES OF COMPUTATIONAL EXPERIMENT: UC MODEL TO BE SIMULATED

The state space model consists of :

Mesurement equation :

$$y_t = \mu_t + \sum_{j=1}^{k} \delta_j x_{jt} + \omega_t + \varepsilon_t$$

State equation :

$$\mu_t = \mu_{t-1} + \eta_t$$

where ε_t and η_t are i.i.d $(0, \sigma_{\varepsilon_t})$ and $(0, \sigma_{\eta_t})$ respectively. And ω_t is an intervention variable

$$\omega_t = \begin{cases} 1 \text{ if } t < \tau \\ 0 \text{ if } t \geq \tau \end{cases}$$

Table 2. Input factors and the UCM estimators obtained from empiriacal work

UCM PARAMETERS –INPUT FACTORS		ESTIMATORS (*t*-value)
REGRESSORS	δ_{PARSER}	**0.97 (4.3)**
	δ_{CONALC}	**-0.24 (-2.43)**
	δ_{SUSP}	**-0.38 (-6.83)***
	δ_{DLAB}	**-0.31 (-2.63)**
	δ_{COMTOT}	**2.03 (4.3)**
TC VARIANCE	σ_{η_t}	**0.02**

Simulation model using the parameters in Table 2.:

$$y_t = \mu_t + \delta_{PARSER}\, x_{1t} + \delta_{CONALC}\, x_{2t} + \delta_{SUSP}\, x_{3t} + \delta_{DLAB}\, x_{4t} + \\ + \delta_{COMTOT}\, x_{5t} + \omega_{LCP} + \varepsilon_t$$

$$\omega_t = \begin{cases} 1 \text{ if } t < 96 \\ 0 \text{ if } t \geq 96 \end{cases}$$

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STAGES OF COMPUTATIONAL EXPERIMENT: DRAG MODEL TO BE ESTIMATED

The general DRAG model is specified as follows :

$$Y_t^{(\lambda_y)} = \sum_{k=1}^{K} \beta_k X_t^{(\lambda_{xk})} + u_t$$

where u_t follows and $AR(l)$ process :

$$u_t = \sum_{l=1}^{r} \rho_l \, u_{t-l} + w_t$$

and the model variables are BCT :

$$Y_t^{(\lambda_y)} = \begin{cases} \dfrac{Y_t^{\lambda_y} - 1}{\lambda_y}, \text{if } \lambda_y \neq 0 \\ \ln(Y_t), \text{if } \lambda_y = 0 \end{cases}$$

Table 3. Response parameters

DRAG PARAMETERS	
REGRESSORS	β_{PARSER}
	β_{CONALC}
	β_{SUSP}
	β_{DLAB}
	β_{COMTOT}
AR PARAMETERS	ρ_1
	ρ_2
BCT COEFFICIENT	λ

- ❑ The DRAG model is estimated using TRIO which is devoloped by Lassarre and Gaudry ()
- ❑ DRAG parameters were estimated using :
  - ❑ Same independent variables as in UC;
  - ❑ Errors follow an autoregressive model of order 2.
- ❑ 8 response parameters in total

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STAGES OF COMPUTATIONAL EXPERIMENT: EXPERIMENTAL DESIGN

Figure1: Experimental design

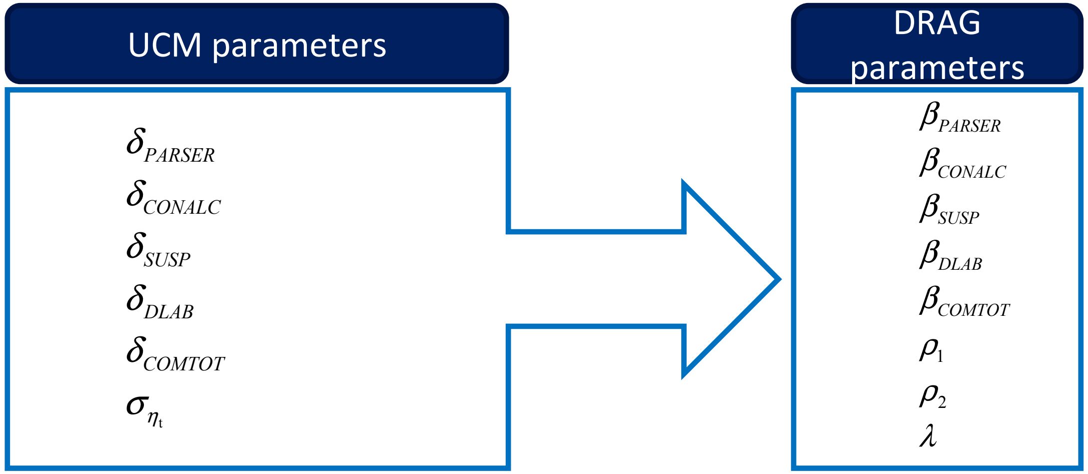

Table 4. Design matrix "+" and "–" for high and low levels of variables.

	δ_{PARSER}	δ_{CONALC}	δ_{SUSP}	δ_{DLAB}	…	
o	-	-	-	-	…	-0.341
δ_{PARSER}	+	-	-	-	…	-0.341
δ_{CONALC}	-	+	-	-	…	-0.306
δ_{SUSP}	-	-	+	-	…	-0.341
δ_{DLAB}	-	-	-	+	…	-0.341
…	…	…	…	…	…	…

STAGES OF COMPUTATIONAL EXPERIMENT: ANOVA ANALYSIS

- Estimation of main effects and interactions (2^{nd}, 3^{rd}, 4^{th}, 5^{th} and 6^{th} order) with the Yates algorithm:

$$\overline{\delta_1} = \frac{1}{4n}\left(-1 + \delta_1 - \delta_2 - \delta_3 - \delta_4 - \ldots\right)$$

$$\overline{\delta_1\delta_2} = \frac{1}{4n}\left(1 - \delta_1 - \delta_2 + \delta_3 + \ldots + \delta_1\delta_2 + \ldots\right)$$

- Computation of ANOVA - sums of squares

$$SS_{\delta_1} = \frac{1}{8n}\left(-1 + \delta_1 - \delta_2 - \delta_3 - \delta_4 + \ldots\right)^2$$

$$SS_{\delta_1\delta_2} = \frac{1}{8n}\left(1 - \delta_1 - \delta_2 + \delta_3 + \ldots + \delta_1\delta_2 + \ldots\right)^2$$

- ANOVA F-tests were applied to test their significance.
- We neglect interactions of 5^{th} and 6^{th} order, to estimate the error variance.

RESULTS AND INTERPRETATION

Table 5. Experimental design results for β_{CONALC} (considering up to 4th interaction effects only)

Variable	Effect estimate	Sum of squares	DF	Mean square	*F*-test	*P*-value
σ_{η_t}	0.1399	0.3133	1	0.3133	72.88889	<0.0001
$\delta_1\delta_{CONALC}\ \delta_3\delta_5$	-0.1216	0.2364	1	0.2364	55.00376	<0.0001
δ_{CONALC}	0.0919	0.1351	1	0.1351	31.41867	0.0001
…	…	…	…	…	…	…
Error	0.0301		7			
Total	1.4820		63			
R^2	0.9797					

- This model explains 97% of the variability in the total model, *F*-test is significan.
- For response β_{CONALC} ,the most significant effects and interactions:
  - ✓ Trend variance, (σ_{η_t})
  - ✓ Second UCM regression coefficient, (δ_{CONALC})

CONCLUSIONS

- Results are as expected intuitively, the trend effect has more complex interpretation. The DRAG regression coefficient is capturing :
  - ✓ the change in the corresponding UCM coefficient,
  - ✓ as well as the change in the variance of the trend component!

- Future research:
  - Effect on prediction errors
  - MANOVA instead of individual ANOVAs
  - Regression models including only significant effects and interactions.
  - In deterministic versus stochastic trend analysis

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